[This question paper contains 4 printed pages.]

6609 *Your Roll No...........*

B.Sc. (Hons.) Computer Science / II Sem. **B**

Paper CS-204 – PROBABILITY

(Admissions of 2001 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *questions are compulsory.*
All *questions carry equal marks.*

1. State Baye's Theorem.
   There are three coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. When one of the three coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin ?

2. Show with the help of an example that pairwise independence does not imply mutual independence (for the events).

3. At a party N men throw their hats into the center of a room. The hats are mixed up and each man randomly select one, Find the expected number of men who select their own hats.

4. The lifetime of a special type of battery is a random variable with mean 40 hours and standard deviation 20 hours. A battery is used unit it fails, at which point it is replaced by a new one. Assuming a stockpile of 25 such batteries, the lifetimes of which are independent, approximate the probability that over 1100 hours of use can be obtained.

5. Let the probability density of x be given by

$$f(x) = \begin{cases} C\left(4x - 2x^2\right), & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) What is the value of C ?

(b) $P\left\{\frac{1}{2} < x < \frac{3}{2}\right\}$

6. Let $X_1, X_2, ---, X_{10}$ be independent poisson random variables with mean 1.

(i) Use the Markov's inequality to get a bound for $P(X_1 + --- + X_{10} \geq 15)$.

(ii) Use the central limit theorem to approximate $P(X_1 + --- + X_{10} \geq 15)$.

7. Stating clearly the appropriate conditions, show that Poisson distribution can be derived as a limiting case of Binomial distribution by using the Stirling's approximation.

8. Suppose that two teams are playing a series of games, each of which is independently won by team A with probability p and by team B with probability (1 – p). The Winner of the series is the first team to win A games. Find the expected number of games that are played.

9. A miner is trapped in a mine containing three doors. The first door leads to a tunnel which takes him to safety after 2 hours of travel. The second door leads to a tunnel which returns him to the mine after 3 hours of travel. The third door leads to the tunnel which returns him to his mine after 5 hours. Assuming that the miner is at all times equally likely to choose anyone fo the doors, what is the approximate length of time until the miner reaches safety ?

10. The joint density of X and Y is

$$f(x, y) = \frac{\left(y^2 - x^2\right)}{8} e^{-y}, \quad 0 < y < \infty, \ -y < x \leq y.$$

show that $E(X/Y = y) = 0$.

11. If X and Y are independent binomial random variables with identical parameters n and p, calculate the conditional probability mass function of X given that $X + Y = m$.

12. Let $\{X_n, n \geq 0\}$ be a Markov Chain having State space $S = \{1, 2, 3, 4\}$ and transition matrix

$$P = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Show that states 1 and 2 are ergodic.

13. On any given day Gary is either cheerful(C), so-so(S) or glum(G). If he is cheerful today, then he will be C, S or G tomorrow with respective probabilities 0.5, 0.4, 0.1. If he is feeling so-so today, then he will be C, S or G tomorrow with respective probabilities 0.3, 0.4, 0.3. If he is glum today, then he will be C, S or G tomorrow with respective probabilities 0.2, 0.3, 0.5. In the long run, what proportion of time is the process in each of the three states ?

14. Explain the Box-Muller approach to simulate independent unit normal random variables from the joint density function :

$$f(x, y) = \frac{1}{2\pi} e^{-\frac{1}{2}(x^2+y^2)}$$

$$-\infty < x < \infty, \quad -\infty < y < \infty$$

15. A joint ensemble $X \times Y$ is generated in the following manner : The X ensemble is $(x_1, 0.4)$, $(x_2, 0.6)$ and Y ensemble is found as follows. If x_1 occurs, then with probability 0.9 y_1 occurs and with probability 0.1 y_2 occurs. If x_2 occurs, then with probability 0.2 y_1 occurs and with probability 0.8 y_2 occurs. Calculate the quantities.

$H(X)$, $H(Y)$, $H(X, Y)$, $H(X/Y)$, $H(Y/X)$, $I(X,Y)$.

Where symbols have their usual meaning.

(300)****